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structured fish populations**

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# Optimal harvesting of age-structured fish populations\*

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## Abstract

This study analyses optimal harvesting of age-structured fish populations and applies a model that can be viewed as a generalization of the surplus production approach. Excluding coincidences, the steady states for the age-structured and the biomass models are equivalent only under zero rate of interest and when the price of fish is independent of fish age and size. For populations with many age classes, the biomass approach may give an "optimal extinction" result under too low a rate of interest. The optimal transition paths are smooth and monotonic for the surplus production approach, while they typically contain oscillations in the age-structured framework. When the age structure is included, optimal harvesting is not based on constant escapement and the optimal yield may decrease in biomass if the age distribution is weighted toward young age classes. In the case of knife edge selectivity, the biomass-sustainable yield curve does not exist in the usual sense and the optimal steady state may be (locally) independent of the rate of discount. With a wide range of parameter values a transition toward smooth sustainable fishing is optimal, although optimal solutions may also exhibit limit cycles and pulse fishing properties.

Keywords: *age-structured models, optimal harvesting, fisheries*

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## 1 Introduction

Economic research on renewable resources and especially on fisheries has applied extensively the surplus production approach for describing the development of harvested populations over time. This has led to models that are suitable for analytical methods and many extensions, such as the game theoretical analysis of open access and fishery regulation. In spite of this success, several authors have questioned whether economic research has offered valuable insights concerning how fish populations should be harvested as dynamically evolving resources. The issue has already been raised by Clark [7,8] who emphasizes that although the "lumped parameter" approach is capable of yielding many valuable bioeconomic insights, it is too simplistic for management purposes. Wilen [35,36] has also noted that the biomass approach<sup>1</sup> is too simple and may at its best serve only as a pedagogical tool. An obvious alternative is the age-structured population model that is extensively applied in fishery ecology and management. Compared to the critical remarks concerning the biomass approach, it is surprising that in his survey Brown [4] found that economists have not judged age-structured models sufficiently useful to compensate for the increased difficulties.

The present study shows that the age-structured model can be viewed as a generalization of the surplus production approach. The generalization reveals hidden assumptions that are included in the surplus production model and shows how adding the population age-structure causes major qualitative changes in the optimal harvesting solutions in addition to the fact that the age-structure information alters the long run optimal steady state harvest and population levels.

The intensive use of the surplus production model in resource economics has also been discussed in fisheries ecology studies. In a widely used text, Hilborn and Walters [15] write that the surplus production model has a bad reputation and it is seen to be a poor cousin of the age-structured analysis. Recently Walters and Martell [34] have written that biologists should base their optimization studies on more realistic age-structured models instead of the surplus production approach.

Townsend [33] investigates studies that have applied the surplus production approach to the American lobster fishery. He concludes that the approach has its pedagogical value in demonstrating how biological and economic factors interact to create a stock externality, but emphasized that simplifications inherent in this modelling may be too great to warrant empirical applications.

Studies on optimal harvesting of age-structured fish populations have been published both in fishery ecology and economics. Getz and Haight [11] presented an extensive survey of age-structured population models and optimal harvesting. However, on closer inspection it turns out that these studies solve the model under more or less *ad hoc* types of restrictions, such as requiring that the harvest is constant over time. From the economic point of view, such an approach may be found to be somewhat inconvenient and there is a clear need to specify age-structured models with a sound economic basis.

Quinn and Deriso [26] address models on optimal harvesting in their

book on fisheries ecology. A large part of their analysis is based on the biomass approach. With respect to optimization models that include an age structure, they refer to studies on suboptimal harvesting policies. One typical example is the study by Hightower and Lenarz [17], where the aim is to find a harvesting strategy that is linear in biomass and that maximizes the average yield over time. Another line of optimization studies [9] aims to develop the Beverton and Holt [2] yield per recruit theory and has produced a collection of biological reference points (such as the  $F_{0.1}$  strategy) that from the economic point of view are rather similar to maximum sustainable yield.

Horwood and Whittle [18] take the original Beverton and Holt [2] model with endogenous recruitment, and maximize discounted net revenue. They study the model numerically using dynamic programming. An approximation is found by computing the optimal steady state and then linearizing the optimality conditions. The resulting solution approximates the true optimal solution in the vicinity of steady state. Assuming that this method yields a solution, it specifies effort as a linear function of the number of fish in different age classes. For cases where an optimal linear control could not be found, the authors expect that the solution has the properties of pulse fishing. This is solved using a different solution method [19], which the authors find somewhat inconvenient. Horwood [20] applies somewhat different linearization methods, but the results are similar.

In spite of the criticism toward the surplus production approach, it is difficult to find clear results from the fishery ecology research that show the properties of optimal harvesting of age-structured fish populations without *ad hoc* -type of simplifications. From the economic point of view, the suboptimal harvesting strategies based on constant harvest or some function of biomass are restrictive. The studies by Horwood and Whittle [18,19,20] retain much more of the original flavor of the problem, but linearization prevents analysis of problems with initial states further from a long run equilibrium. Beddington and Taylor [1], Reed [28] Getz [10], Brooks [3] and several others have studied optimal sustainable harvesting using an age-structured model and the assumption that harvesting is perfectly selective among age-classes. The optimal (MSY) steady state solution is bimodal in the sense that only one or two age classes are harvested. These results are clear, but the perfect selectivity assumption has somewhat limited relevance in the case of fisheries.

An early fishery economic application of the classical Beverton and Holt [2] model is Hannesson [14]. He found (numerically) that the optimal solution for the age-structured model has a pulse fishing property. Clark [8] writes that this framework is vastly more complex than the Schaefer [30] surplus production approach, and that in its general form the problem is almost incomprehensible. Under simplifications such as exogenous recruitment and linear objective, Clark found that the optimal solution almost inevitably follows the pulse fishing strategy. This result is somewhat inconvenient since a smooth sustainable yield and stable income are typically included as the important targets of any successful fishery policy. A recent study by Stage [31] applies an age-structured model for Namibian linefishing and finds that the relative profitability of recreational and

commercial fisheries depends on the length of the planning horizon. He calls for more economic research on age-structured models.

In spite of the studies referred to above, the properties of optimal harvesting of age-structured fish populations are incompletely understood. It is quite clear that there exists two opposite schools, one favoring the surplus production and the other the age-structured model. It would therefore be constructive to obtain a better understanding of how the inclusion of the age-structured information changes the qualitative properties of optimal harvesting from those obtained under the surplus production model. Two recent studies approach this question. Moxnes [24] compares suboptimal strategies<sup>2</sup> for harvesting an age-structured population and the solutions from the surplus production model for the same fishery. He finds that the differences between these two approaches are rather minor. Tahvonen [32] studies the outcomes of applying the feedback solutions from the surplus production model for harvesting a population that is actually age-structured. The results show that the performance of the surplus production model depends critically on the initial age distribution. Under the existence of multiple steady states, neglecting the age-structure information may yield accidental depletions and developments toward different steady states than expected. However, that study does not attempt to present optimal harvesting solutions for the age-structured model.

The present study applies the generic age-structured population model from the fisheries ecology literature [15] with endogenous nonlinear recruitment. The harvest is assumed to occur in the middle of each period. This makes the age-structured model more amenable to economic analysis than the original Beverton and Holt [2] formulation where effort is constant over each period. Parameter values are for the Atlantic menhaden fishery. It is shown how the surplus production model can be obtained from the age-structured model under equilibrium conditions. This connection between the models forms a basis for theoretical comparisons and shows how the age-structured model can be viewed as a generalization of the surplus production approach. The optimal solutions are computed applying recently developed methods for large scale nonlinear programming and without applying linearization or any *ad hoc* constraints.

For the surplus production model, the optimal steady state is determined by the equality of the rate of interest and the derivative of the natural production function (when harvesting costs are absent). Under linearity assumptions, the optimal solution has the constant escapement property. Under nonlinearities, the optimal yield is an increasing function of biomass. Transient solutions are monotonic paths toward a steady state. This study shows that there is no guarantee that any of these properties will carry over to the age-structured framework. Excluding coincidences, the steady states for the age-structured and the biomass models are equivalent only under zero rate of interest and when the price of fish is independent of fish age and size. For populations with many age classes, the biomass model may give an "optimal extinction" result under too low a rate of interest. The optimal transition paths of the two models have different properties qualitatively and fluctuations are typical for the age-structured model but do not exist within the biomass approach.

In the age-structured model, optimal harvesting is never based on constant escapement and the optimal yield may decrease in biomass if the age-structure is weighted toward young age-classes. In the case of knife edge selectivity, the biomass-sustainable yield curve does not exist in the usual sense and the optimal steady state may be independent of discounting. With a wide range of parameter values, a transition toward smooth sustainable fishing is optimal, although solutions may also exhibit limit cycles and pulse fishing properties.

These results, and the optimization model specification applied, are new both in fishery economics and ecology. From the economic point of view, the study emphasizes that the population age-structure includes valuable information on future harvesting possibilities that is neglected when the surplus production model is applied.

The paper is organized as follows. Section 2 introduces the age-structured fishery model and the economic optimization problem. The next section presents steady state results and section five the dynamic analysis. The results are summarized in the final section.

## 2 The optimization problem

This study is based on a generic age-structured population model from the fishery ecology literature [15,11,26]. In these models, harvest may occur instantaneously at the beginning, middle or at the end of each period [25,26,34,36] or simultaneously with natural mortality at a constant rate as in Beverton and Holt [2]. Clearly, the choice between different alternatives should be based on the empirical facts of the given fishery under study. In the model applied here, fishing is assumed to occur instantaneously at the middle of each period.<sup>3</sup> One reason behind this choice is that it enables the model to be studied in a form that is linear in effort and yield and that can be compared with the well-known similar specification of the surplus production model with the constant escapement (or bang-bang) solution.

Let  $x_{st}$ ,  $s = 1, \dots, n$ ,  $t = 0, 1, \dots$  denote the number of fish of age class  $s$  at the beginning of period  $t$ . The number of eggs (or newborns) is denoted by  $x_{0t}$ . Let  $f_s$ ,  $s = 1, \dots, n$  denote the fecundity parameters. The number of eggs is given as

$$x_{0t} = \sum_{s=1}^n f_s x_{st}. \quad (1)$$

Thus, spawning also occurs in the beginning of each period. Only a fraction of eggs will survive as recruits. When  $\phi$  denotes a recruitment function, the next period number of recruits is given as

$$x_{1,t+1} = \phi(x_{0t}). \quad (2)$$

Let  $m$  denote the rate of natural mortality within each period, and assume that it is equal<sup>4</sup> for all the age classes. If fishing mortality is zero, a fraction  $e^{-m}$  of an age class will survive for the next period. After half a year, the fraction that is alive equals  $e^{-m/2}$ . Assuming that fishing occurs in the middle of each

period implies that the development of the number of fish in each age class, excluding age class 1 and  $n$ , can be written as

$$x_{s+1,t+1} = e^{-m/2}(e^{-m/2}x_{st} - h_{st}), \quad s = 1, \dots, n-2, \quad t = 0, 1, \dots, \quad (3)$$

where  $h_{st}$ ,  $s = 1, \dots, n-2$ ,  $t = 0, 1, \dots$  denote the number of fish harvested. Next, the development of the age class  $n$  (including all the older age classes) is given as

$$x_{n,t+1} = e^{-m/2}(e^{-m/2}x_{n-1,t} - h_{n-1,t}) + e^{-m/2}(e^{-m/2}x_{nt} - h_{st}), \quad t = 0, 1, \dots \quad (4)$$

Age-structured fishery models and Virtual Population Analysis [15,12] are heavily based on the Schaefer [30] production function. However, as written e.g. by Clark [8] this production function is rather restrictive and more generally the age-class specific harvest may be written as  $h_{st} = Q_s(E_t, e^{-m/2}x_{st})$ ,  $s = 1, \dots, n$ ,  $t = 0, 1, \dots$ , where  $E_t$  is effort and  $Q_s$  is the production function that is increasing in effort and the number of fish (at the moment of harvest). Note that under this specification different age classes cannot be harvested independently, i.e. effort is nonselective. Obviously, the number of harvested fish cannot exceed the number that exists in the given age class at the moment of harvesting, i.e.  $h_{st} \leq e^{-m/2}x_{st}$ ,  $s = 1, \dots, n$ ,  $t = 0, 1, \dots$ . A straightforward application of these restrictions would imply a constraint on effort. However, the number of fish in any age class restricts the number of fish caught, but not the number of vessel weeks at sea. To take this fact into account requires an additional set of restrictions of the form  $h_{st} = e^{-m/2}x_{st}$ , if  $Q_s(E_t, x_{st}) \geq x_{st}$ ,  $s = 1, \dots, n$ ,  $t = 0, 1, \dots$ . These restrictions can be formulated as the following complementary constraints:

$$h_{st} - Q_s(E_t, e^{-m/2}x_{st}) + y_{st}^1 = 0, \quad s = 1, \dots, n, \quad t = 0, 1, \dots \quad (5)$$

$$h_{st} - e^{-m/2}x_{st} + y_{st}^2 = 0, \quad s = 1, \dots, n, \quad t = 0, 1, \dots \quad (6)$$

$$y_{st}^1 \geq 0, \quad y_{st}^2 \geq 0, \quad y_{st}^1 y_{st}^2 = 0, \quad s = 1, \dots, n, \quad t = 0, 1, \dots, \quad (7)$$

where  $y_{st}^i$ ,  $i = 1, 2$ ,  $s = 1, \dots, n$ ,  $t = 0, 1, \dots$  are complementary or slack variables.

The weight of fish increases when they reach a new age class and the weight of age-class  $s$  fish is given by  $w_s$ ,  $s = 1, \dots, n$ . The total (physical) yield  $Y_t^P$  is obtained by summing the yield over the age classes and equals

$$Y_t^P = \sum_{s=1}^n w_s h_{st}, \quad t = 0, 1, \dots \quad (8)$$

Consequently, the total biomass  $B_t$  of the population equals

$$B_t = \sum_{s=1}^n w_s x_{st}, \quad t = 0, 1, \dots \quad (9)$$

In addition to specification (8), it is possible to take into account that the price of fish may depend on fish age and size. For some species, price may increase in size e.g. due to easier mechanical treatment of harvested fish. For some other species, price may decrease with size, and age due to deterioration of quality of the reproductive age classes and due to increasing residues contamination, for example. Assuming that price depends on fish age or size the total revenues from an annual harvest are given by

$$Y_t^R = \sum_{s=1}^n p_s w_s h_{st}, \quad t = 0, 1, \dots, \quad (10)$$

where  $Y_t^R$  denotes revenues and  $p_s$ ,  $s = 1, \dots, n$  the price for each age (or size) classes of fish.

Assume that  $U$  is an increasing and concave utility function and  $C$  an increasing and convex cost function for effort, respectively. Given that  $V$  is the economic value of the population and  $b = 1/(1+r)$  the discount factor ( $r$  is the rate of interest), the objective function of the optimal harvesting problem is

$$V(\mathbf{x}_0) = \sum_{t=0}^{\infty} b^t [U(Y_t^i) - C(E_t)], \quad (11)$$

where  $\mathbf{x}_t$  is the vector for the initial age class distribution and  $Y_t^i$ ,  $i = P, R$  may denote the physical yield or economic revenues. The problem of the optimal harvesting of the age structured population can now be defined as the problem of choosing a time path for effort  $E_t$  in order to maximize  $V(\mathbf{x}_0)$  subject to restrictions (1)-(8) and subject to the following initial and boundary conditions

$$x_{s0}, s = 1, \dots, n, \text{ given and} \quad (12)$$

$$E_t \geq 0, \quad t = 0, 1, \dots \quad (13)$$

### 3 Optimization procedure and data on population growth

The model defined by (1)-(13) may be viewed as a large-scale nonlinear programming problem with complementary (or equilibrium) constraints. For the numerical solutions, this study employs Knitro optimization software [5,6] that includes state-of-the-art interior (or barrier) and active-set methods. When the interior point method is applied, the solver may use either the iterative Conjugate Gradient approach or it may factor the Karush-Kuhn-Tucker (primal-dual) matrix directly. The system has been evaluated extensively [37] and it is suitable for smooth problems but does not require convexity. In addition, it applies specialized methods for complementary constraints [22]. It is possible to choose the initial guesses by using a randomized multi-start procedure when seeking the globally optimal solution .

For the purposes of numerical analysis, this study utilizes data that has been collected and estimated in fishery stock assessment research. However, the aim of this study is not to present results on how some particular fishery should be managed. Instead, the aim is to investigate the basic theoretical

features of optimizing the harvest from age-structured fish populations. Table 1 presents data for the Atlantic menhaden fishery from a study by Hightower and Grossman [16] (see also [11]). This population is described by eight age classes. Natural mortality is 0.25 and equal for all age classes<sup>4</sup>. For recruitment Hightower and Grossman [16] applies the Ricker [29] recruitment function:

$$x_{1,t+1} = x_{0t}\alpha e^{-\beta x_{0t}}, \quad (14)$$

where  $\alpha = 0.0205$  and  $\beta = 0.0024$ . Note that the catchability coefficients do not increase monotonically with fish age and size. This is not at all exceptional and reflects the properties of the fishing technology [23,12].

Age (yr)	Weight (g)	Fecundity (g)	Catchability
1	102.77	0	0.0577
2	260.21	110.25	0.1805
3	411.73	227.37	0.1579
4	530.31	302.70	0.1540
5	614.28	354.63	0.1430
6	670.60	410.44	0.1820
7	707.23	491.98	0.1703
8+	730.63	469.6	0.1703

Table 1. Age class data for the Atlantic menhaden fishery [16]

#### 4 Steady state analysis and some comparisons with the surplus production model

For populations with an age class structure, the surplus production model can be considered to represent an equilibrium solution for the age-structured model. Lawson and Hilborn [21] write that "whenever the surplus production model is desired, the best way to calculate the parameters may be to use the age-structured model". For this purpose let  $x_{s\infty}$ ,  $s = 1, \dots, n$  denote the equilibrium number of individuals and take effort as a constant. Assuming the Schaefer [30] production functions  $h_{st} = q_s x_{st} E_t$ ,  $s = 1, \dots, n$ , where  $q_s$ ,  $s = 1, \dots, n$  are catchability coefficients it is possible to write equations (3)-(4) as

$$x_{s+1,\infty} = x_{s\infty}\mu_s, \quad s = 1, \dots, n-1, \quad \text{where} \quad (15)$$

$$\mu_s = e^{-m}(1 - q_s E), \quad s = 1, \dots, n-2, \quad (15)$$

$$\mu_{n-1} = e^{-m}(1 - q_{n-1}E)/(1 - e^{-m} + e^{-m}q_n E). \quad (16)$$

Note that in equilibrium, the complementary constraints (5)-(7) are satisfied in a form  $y_s^1 = 0$ ,  $y_s^2 \geq 0$  for  $s = 1, \dots, n$  implying that  $h_s = q_s x_s E$ . Next, using (15)-(16), it is possible to write the equilibrium age-class structure for  $s = 2, \dots, n$  in terms of  $x_{1\infty}$ :

$$x_{s\infty} = \Phi_s x_{1\infty}, \quad s = 2, \dots, n, \quad \text{where} \quad (17)$$

$$\Phi_s = \prod_{i=1}^{s-1} \mu_i, \quad s = 2, \dots, n. \quad (18)$$

The equilibrium number of newborns (or eggs) can now be written as

$$x_{0\infty} = x_{1\infty} \sum_{s=1}^n f_s \Phi_s, \quad (19)$$

where  $\Phi_1 \equiv 1$ . Given the Ricker [29] recruitment function and the definition  $\sum_{s=1}^n f_s \Phi_s \equiv R$  it is possible to write the remaining equation as

$$x_{1\infty} = x_{1\infty} R \alpha e^{-\beta x_{1\infty} R}, \quad (20)$$

which can be solved for  $x_{1\infty}$  :

$$x_{1\infty} = \frac{LN(R\alpha)}{R\beta}. \quad (21)$$

Next, applying equations (17)-(18) and the result in (21) yields the equilibrium number of fish over the age classes and equations (8)-(9) the total harvest and biomass respectively. Thus, it is possible to vary the level of biomass and solve the equilibrium effort and sustainable yield. The relationship between biomass and yield represents the surplus production model that corresponds with the age-structured data. Equation (21) shows that a strictly positive equilibrium exists only if  $R\alpha > 1$ . Using the data from Table 1 yields  $R\alpha \simeq 22$ , for  $E = 0$ . Reed [28] has shown that an equilibrium of the age-class model is locally stable if  $|R\partial\phi(x_{0\infty})/\partial x_{0\infty}| < 1$ . This condition is satisfied for the data in Table 1.

Figures 1 a and b show analysis of the biomass-yield relationships based on the data in Table 1. In Figure 1a, the solid line is the equilibrium yield as a function of biomass. The carrying capacity is about 2316 thousand tons and the maximum sustained yield (MSY) about 536 thousand tons. As became clear in the derivation of equations (15) to (21), this function is an outcome of both the biological properties of the fish population and the fishing technology that is included in the production functions and catchability coefficients. To demonstrate this the lowest function in Figure 1a shows the equilibrium yield if the catchability coefficient of age class 1 is increased to be equal with the coefficient of age class 2. As shown, this decreases sustainable yield over all biomass levels. The highest function in Figure 1a shows the equilibrium yield if a perfectly selective technology could be used. To maximize the equilibrium yield at a given biomass level typically requires harvesting fish from only one or two age classes. The circles show biomass levels where the optimal harvesting regime switches between the age classes. Below biomass level 840 it is optimal to harvest age classes 1 and 2 and, for example, above biomass level 2110 the optimal harvest is aimed at age classes 7 and 8.

Similarly, as harvesting technology determines the physical sustainable yield, the price variation over the age or size of fish may determine the level of sustainable revenues. In Figure 1b, the higher dashed line shows sustainable revenues (catchability coefficients from Table 1) if the price of fish increases with age and size ( $p_s = 22.8 + 2.8 \times s$ ). The lower dashed line shows the case where the market price decreases with age and size ( $p_s = 22.8 - 2.8 \times s$ ). As shown, the maximum sustainable revenues depend on the price structure and

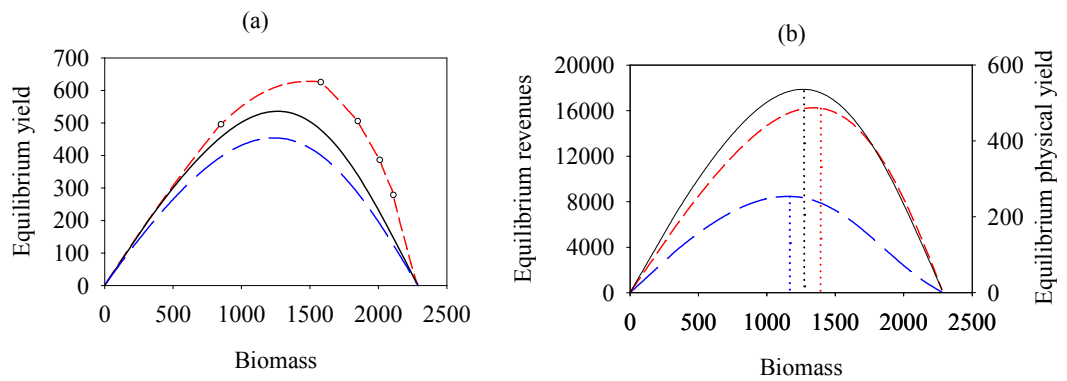


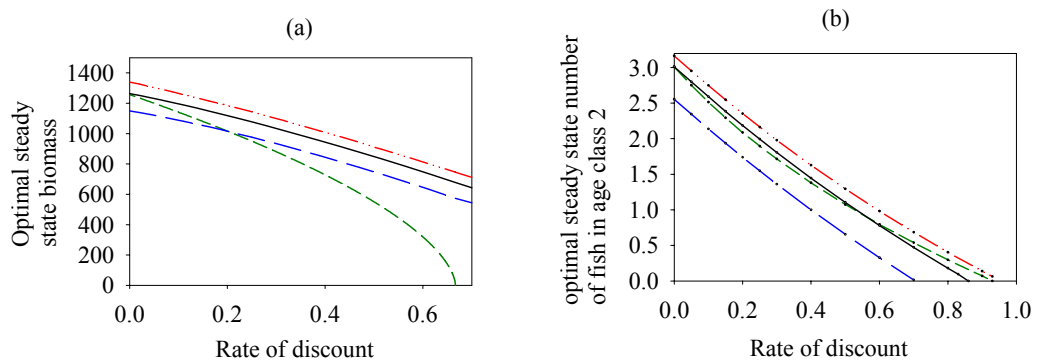
Figure 1a,b. Effects of fishing technology and fish price on equilibrium yield and revenues

(a): Dependence of equilibrium yield on harvesting technology

- Perfect selectivity
- Benchmark catchability in Table 1
- - -  $q_1$  increased to 0.1805 ( $=q_2$ )
- A switch point between harvested age classes under perfect selectivity

(b) Dependence of equilibrium revenues on fish price

- - - Price decreases with size
- Price increases with size
- Benchmark equilibrium yield



Figures 2a and b. Comparison of optimal steady states of the surplus production and age-structured models under zero harvesting cost

(a) Steady states when the model includes all the eight age classes

(b) Steady state number of age class two fish when the model includes two age classes

- - - Biomass model
- Age structured model when price independent of fish size
- · - · - Age structured model when price increases with fish size
- - - Age structured model when price decreases with fish size

may be realized above or below the biomass level that maximizes the physical sustainable yield.

Figures 2a and b show comparisons of optimal steady states between the surplus production and the age-structured models under when harvesting cost is zero. The fact that in the age-structured model sustainable yield depends on the harvesting technology readily implies that, even in the simplest case without harvesting cost, the economically optimal steady state is not determined by purely biological factors and the rate of interest. This should be compared to the surplus production approach where (in the absence of harvesting cost) the optimal steady state is determined by  $dF(B)/dB = r$ , and  $F(B) = H$ , where  $H$  is harvest and  $F$  is interpreted as the natural growth rate of the population. Clearly, from the point of view of the age-structured approach, interpreting  $F$  as a purely biological function is problematic, since it cannot be defined without assumptions on the harvesting technology.<sup>5</sup>

Figure 2a shows results for the model with all the eight age classes. As shown, the steady states with the surplus production model and the age-structured model coincide only when the rate of discount is zero and price is independent of the fish age and size. Under fixed price, the optimal steady state biomass is lower for the surplus production model and for this model the steady state does not exist when  $r \geq 0.86$ . In comparison with the age-structured model, the optimal steady state biomass at this critical rate of interest is about 700 thousand tons and depletion of the population becomes optimal only with some higher rate of interest. In addition, Figure 2a shows how the dependence of price on the age and size of fish changes the optimal steady state in the age-structured model.

Figure 2b shows a similar computation for a reduced model with only the two youngest age classes, i.e. it is assumed that  $n = 2$  in the data of Table 1. The steady state is now given in terms of the number of fish in age class two. The essential difference with the full eight age-classes model is that now the age structured model yields higher steady states with low rate of interest but lower steady states with higher rate of interest compared to the the surplus production model.

A complete interpretation of the factors that cause the deviation between the steady states requires an analytical derivation of the steady state equations for the age-structured model. This may well be possible, but it is somewhat tedious due to the structure determined by the nonselective harvesting technology. However, it is clear that the biomass model cannot capture the fact that when the harvest is reduced to increase the future steady state yield, the return is determined via a time-delay structure through all the age classes. The biomass approach cannot capture these effects and the present value of marginal return as is possible when the age-structured model is used.

## 5 Dynamic analysis

### *Linear specification*

Let the utility and cost functions take the forms  $U(Y_t^p) = Y_t^p$  and  $C(E_t) = cE$ . Figure 3 is based on an example where  $b = 0.97$  and  $c = 10$ . In ad-

dition, assume the Schaefer production functions, i.e.  $Q_s(E_t, e^{-m/2}x_{st}) = q_s E_t e^{-m/2}x_{st}$ ,  $s = 1, \dots, n$ . This implies that the problem is linear with respect to effort and total yield<sup>6</sup>. Thus, the properties of the optimal solution can be compared with the well-known optimal constant escapement policy that is obtained under similar assumptions for the surplus production model [27].

In Figure 3a-b, the optimal solution is studied by choosing 200 initial age distributions randomly and plotting the initial biomass and the first period optimal solution for aggregate yield and escapement. The optimal steady state is denoted by a circle at the point  $e^{-m/2}B_0 = 1124$ ,  $Y_0 = 536$ . In Figure 3a, the dotted line shows the equilibrium yield as a function of biomass (cf. Figure 1a). Figure 3a shows that optimal total yield is not a function of biomass since it depends on how the biomass is distributed over the age classes.

Figure 3b shows the optimal escapement, i.e.  $e^{-m/2}B_0 - Y_0$ . If the initial biomass is small enough, the optimal yield is zero and optimal escapement equals the biomass before the harvest. This solution may be optimal up to  $e^{-m/2}B_0 \simeq 800$  depending on how the initial biomass is distributed over the age classes. When the initial biomass is higher, the optimal escapement tends to decrease. This agrees with Figure 3a, where the optimal total yield tends to increase faster than the total biomass. This feature reflects the fact that both higher catchability coefficients for three oldest age classes and the Schaefer production function imply a higher optimal catch when the population is large, even at the expense of somewhat smaller catch in the next period.

For comparison, the optimal constant escapement strategy is depicted in Figures 3a and b by dashed lines. Since the optimal yield depends on the age class distribution and the optimal yield increases faster than the biomass, the constant escapement strategy deviates quite clearly from the true optimal solution. Note that at the biomass level of 800 thousand tons the optimal escapement may equal 800 thousand tons (i.e. the optimal yield is zero), while at biomass level 2300 the optimal escapement level may be much lower and equal to about 400 thousand tons.

Figures 4a and b show how the optimal solutions proceed over time. In the four examples, the optimal solutions approach the same steady state. This feature also holds for all the 200 examples in Figures 3a and b. Thus, these steady states may be globally stable. Another feature of the optimal solutions is that they may overshoot the steady state several times, again demonstrating the fact that the optimal solution is more complex than the constant escapement policy. However, it is worth noting that even in the case of linear specification the optimal solution approaches a steady state with smooth sustainable harvest instead of pulse fishing (cf. [14]).

The constant escapement strategy is discussed both in fishery economics and ecology literature [8,27,15,26,34]. It has been found to have potential practical relevance due to its simplicity and it has been proposed in the context of both the age-structured and the surplus production approaches. The results shown in Figures 3 and 4 nevertheless show that the policy is not optimal for age structured populations. In addition, if this policy is applied, the steady state will not be reached in such a straightforward manner as expected.

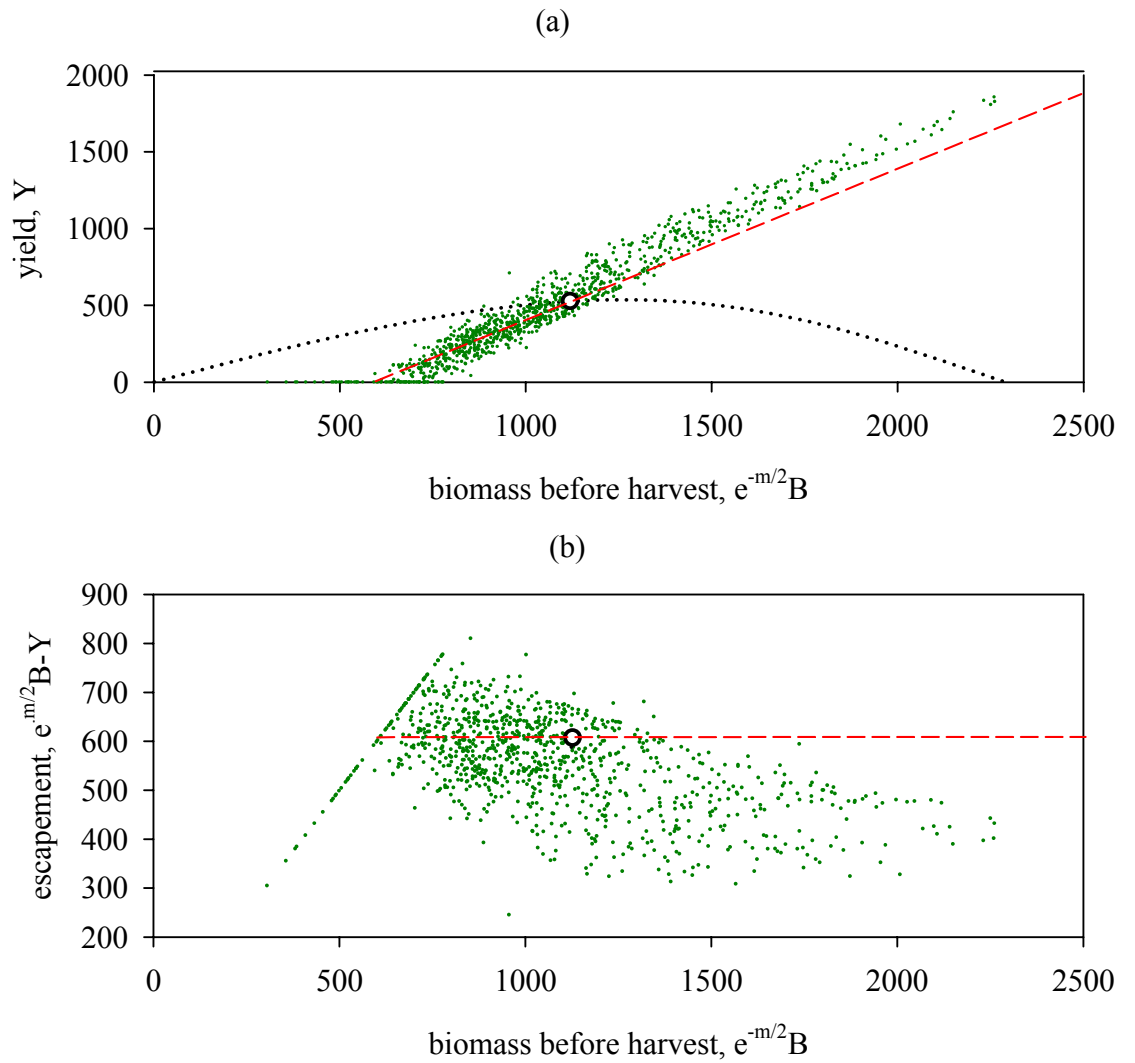


Figure 3 a,b. Effects of age class distribution on optimal escapement and yield

- ..... Equilibrium biomass-yield function
- - - Constant escapement policy
- Optimal first period solution

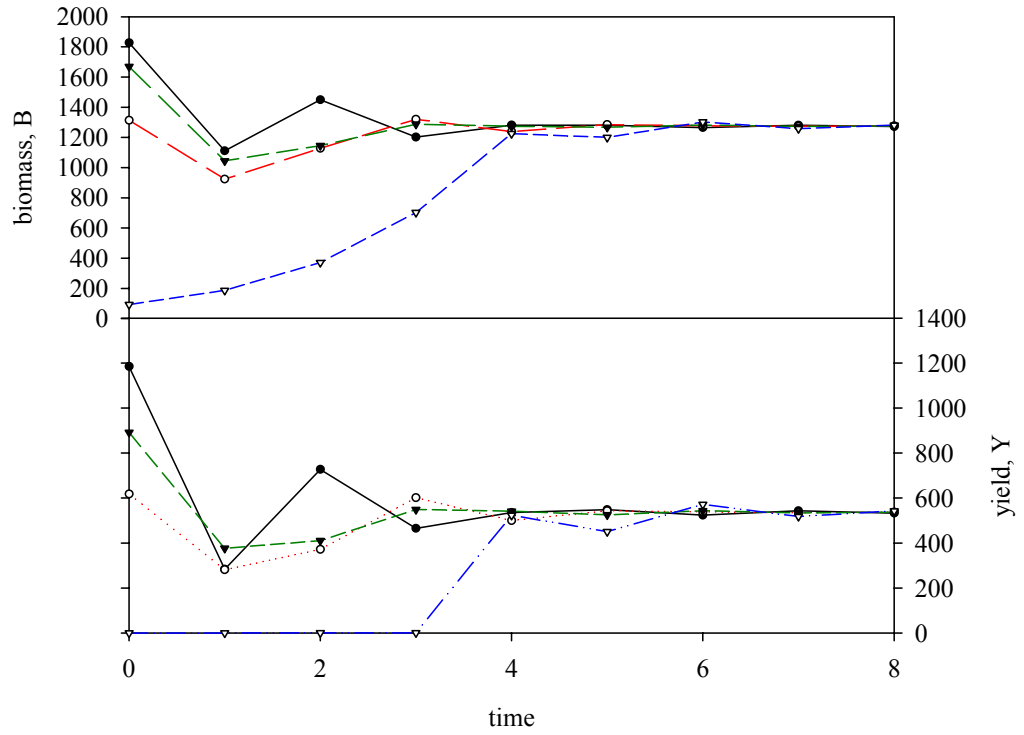


Figure 4. Optimal solutions with different initial states

### *Nonlinear specifications*

To examine the optimal solution under nonlinearities, assume  $U(Y_t) = (10Y_t)^{0.6}$ ,  $C(E_t) = 0$ ,  $b = 0.8$ , and the Schaefer production function where the catchability coefficients are from Table 1. These parameter values<sup>7</sup> imply a steady state where  $B_\infty \simeq 1078$ ,  $Y_\infty = 519$  (Figure 5a). The dotted line shows the equilibrium biomass-yield relationship. Since fishing costs are zero and the rate of discount is positive, the optimal steady state exists below the MSY biomass. Figure 5a shows three examples for optimal solutions that converge toward this steady state. The other solutions in Figure 5a follow from an example where  $U(Y_t) = (10Y_t)^{0.6}$ ,  $C(E_t) = 60E_t^2$ ,  $b = 0.97$  with the production functions the same as before. In this case, the rate of discount is low and fishing is costly which, together with the Schaefer production function, imply that the steady state biomass is higher and the yield is lower than the MSY levels, i.e.  $B_\infty = 2071$ ,  $Y_\infty = 178$ . Figure 5a shows four examples of optimal solutions that converge toward the steady state. As in the case of linear specification (Figure 3 and 4), it is likely that under both parameter values the steady states are globally stable (in the saddle point sense).

A key feature of the optimal solution is that optimal yield is not a function of the biomass but a function of the number of fish in each age class. In spite of this, it is possible to view the examples in Figure 5a as if the optimal yield is higher when the biomass is higher. This relationship may be weaker if, for example, the production functions deviate from the Schaefer formulation. The four examples in Figure 5b are based on the production function  $Q_s(E_t, e^{-m/2}x_{st}) = q_s E_t^{0.9} (e^{-m/2}x_{st})^{0.1}$ . This specification may be suitable for pelagic schooling species and under gear saturation [8,26]. The other parameter values are the same as for the example with the higher steady state biomass in Figure 5a. Figure 5b shows that under gear saturation or pelagic schooling the total yield depends more strongly on the age distribution of the population than in the case of the Schaefer production function. The solutions with low initial yield have an initial age structure where older age classes are close to zero, implying that optimal yield may be low although the population biomass may equal or exceed the steady state level.

Figure 6 shows how the age class structure develops over time. The parameter values are as in Figure 7. After 30 periods, the age structure is close to steady state distribution. The steady state age class structure is a consequence of natural mortality and fishing technology specified in production functions and catchability coefficients. It would however, be possible to add several different fishing technologies into the model with the implication that harvest can be targeted more efficiently to specific age classes.

### *Effects of some bioeconomic parameters*

A fishery may differ considerably from the features specified in Table 1. In this data, all age classes are vulnerable to fishing. However, it is possible that fishing gear has a knife edge selectivity property meaning that some youngest age classes are

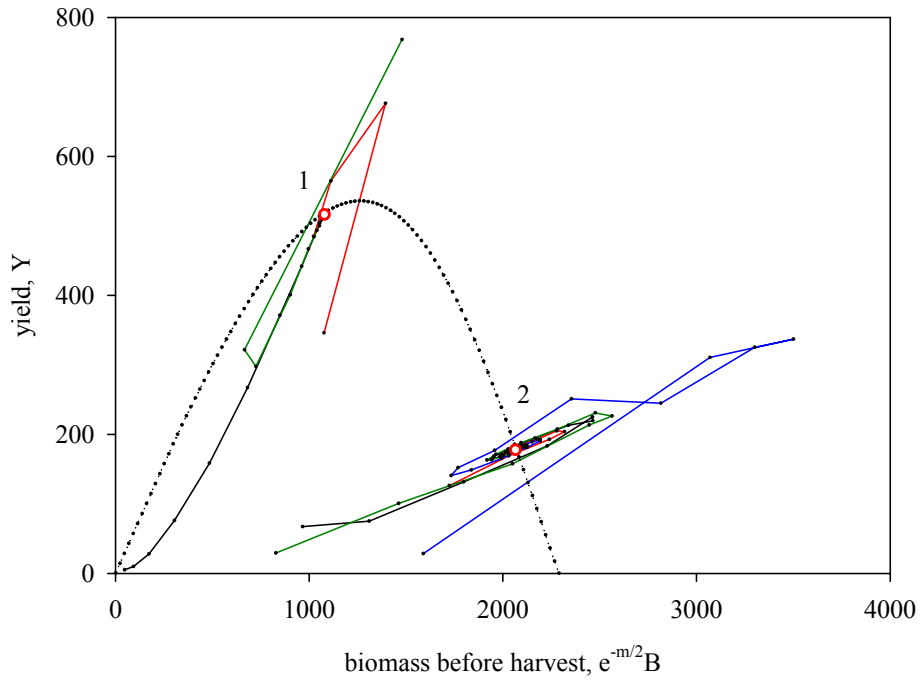


Figure 5a. Optimal solutions under nonlinear utility  
 Steady state 1: High rate of interest, zero harvesting cost  
 Steady state 2: Low rate of interest, high harvesting cost

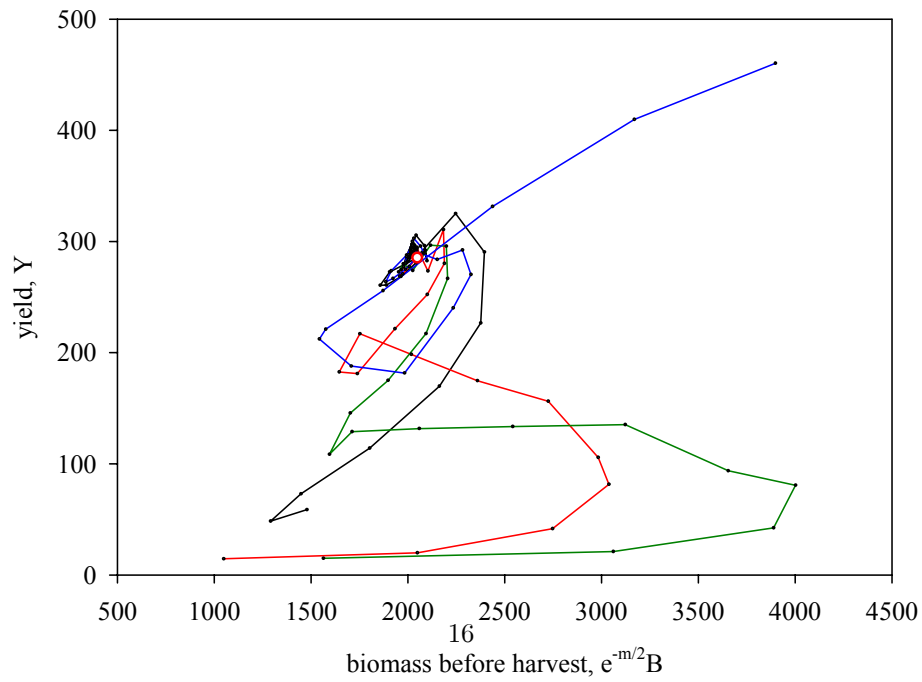
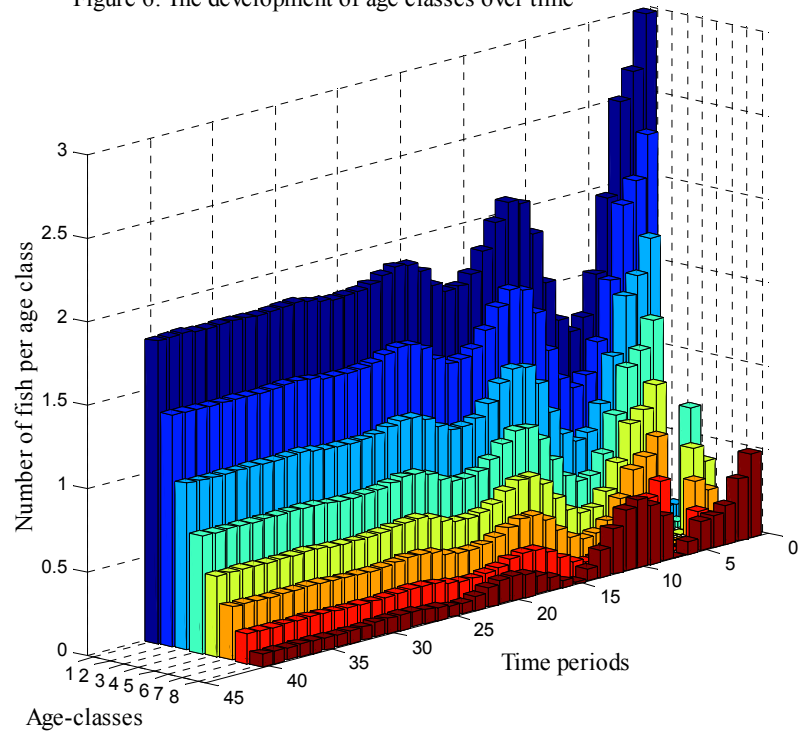


Figure 5b. Optimal solutions under nonlinear utility and gear saturation

Figure 6. The development of age classes over time



left unharvested. Another possibility is that fecundity is zero for youngest age classes. In addition, it is possible that the youngest age classes are commercially valueless simultaneously with nonselective fishing gear. Such changes have rather strong implications for the properties of the model.

Figure 7 shows the baseline case where, in addition to the parameter values in Table 1, it is assumed that the production functions follow the Schaefer specification and  $U(Y_t) = (10Y_t)^{0.7}$ ,  $b = 0.97$ ,  $C(E_t) = 0$ . Two optimal solutions are shown in an aggregate biomass-yield state space. For comparison, the dotted lines show the optimal solutions (feedback control) for the surplus production model. However, it should be noted that simple comparisons of these solutions may be misleading since if the harvested population actually evolves as an age-structured system the development of the population cannot accurately be predicted by applying the aggregated biomass information only [32].

In Figure 8 it is assumed that fishing gear has the knife edge selectivity property. Under such fishing technology, the model properties change drastically. If harvest affects only the six oldest age classes and the level of effort is high enough, in the resulting equilibrium age class three is totally harvested every period and older age classes do not exist. Obviously, it is not possible to reach lower biomass levels independently of the rate of interest. Note that the equilibrium biomass-yield function exist only for biomass levels containing age class three and older fish. In Figure 8 this lowest attainable biomass level is somewhat below 1600 thousand tons. The solution that reaches this biomass level after several overshootings has the same parameter values as the benchmark case in Figure 7 (the solution with higher initial biomass) excluding that  $q_1 = q_2 = 0$ . The other solution in the figure follows if harvesting costs are positive but linear ( $c = 50$ ). The case of knife edge selectivity is highly relevant in fishery regulation but it is unclear how the surplus production approach could describe fisheries applying such technologies. More complications would arise if a possibility to optimize over several fishing technologies is added to the model.

Figure 9a and b describe a case where fecundity is zero for age classes  $s = 1, \dots, 4$ . In this case, the equilibrium biomass-yield function exists as normally but its shape differs from the benchmark case. Again, the optimal solution for the age-structured model contains overshootings, while the expected feedback solutions for the surplus production model are monotonic paths toward the steady state with a slightly lower biomass level. Figure 9b compares the optimal solutions under high interest rates. Zero fecundity for the young age classes decreases the slope of the equilibrium biomass-yield function for low biomass levels implying that the existence of optimal steady states becomes more critical. In the example, an interest rate equal to  $r = 0.287$  is high enough to imply that under the surplus production model it is optimal to deplete the population. However, as shown, an optimal sustainable harvesting solution exists for the age-structured model. Clearly such an interest rate is high but nevertheless the example demonstrates a crucial difference between the surplus production and the age-structured model.

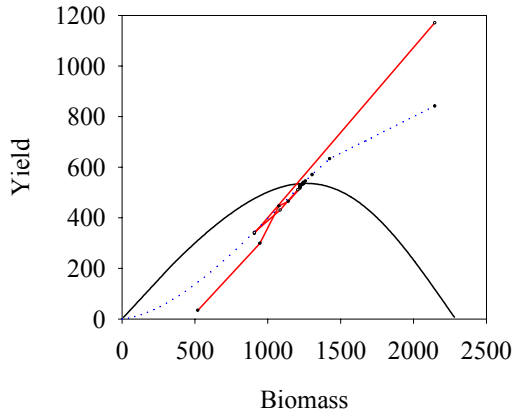


Figure 7. Comparison of surplus production and age-structured models

- Age-structured model
- ⋯ Surplus production model
- Equilibrium yield

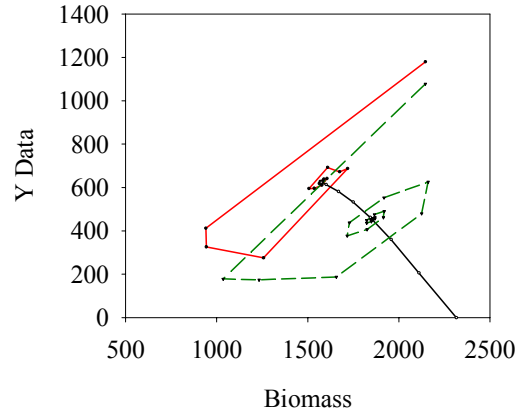


Figure 8. Optimal solutions under knife edge selectivity ( $q_1=q_2=0$ )

- Zero harvesting cost
- - - Positive harvesting cost ( $c=50$ )
- Equilibrium yield

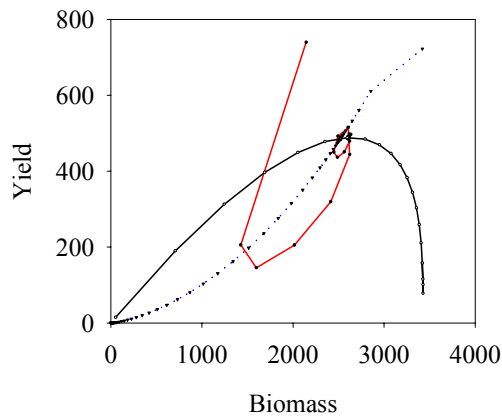


Figure 9a. Zero fecundity for young age classes ( $q_2=0, s=1,2,3,4$ )

- Age structured model
- ⋯ Surplus production model
- Equilibrium yield

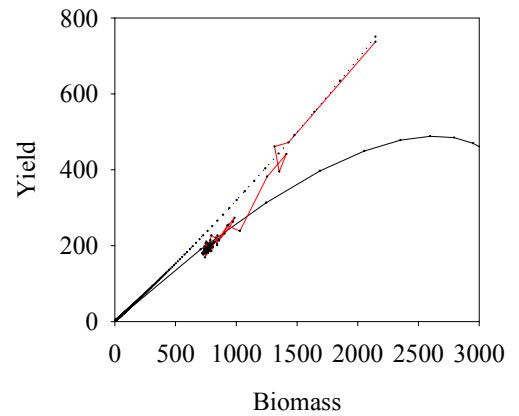


Figure 9b. Zero fecundity for young age classes and high rate of interest: extinction under the biomass model, sustainable harvest under age-structured model

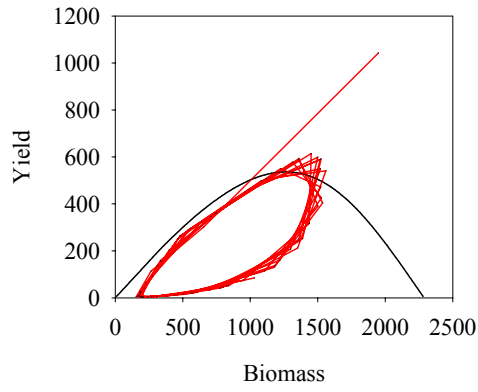


Figure 10a. Optimal limit cycle when the three youngest age classes are commercially valueless

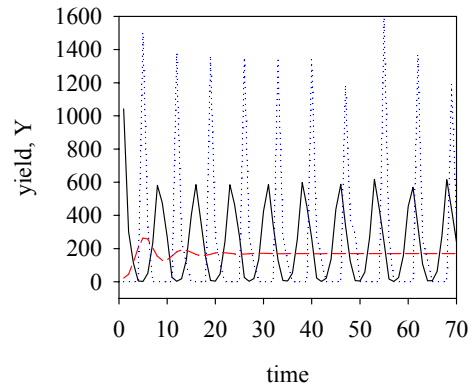


Figure 10b. Optimal pulse fishing when the three youngest age classes are commercially valueless

- ..... linear utility
- moderately concave utility function
- - - strongly concave utility function

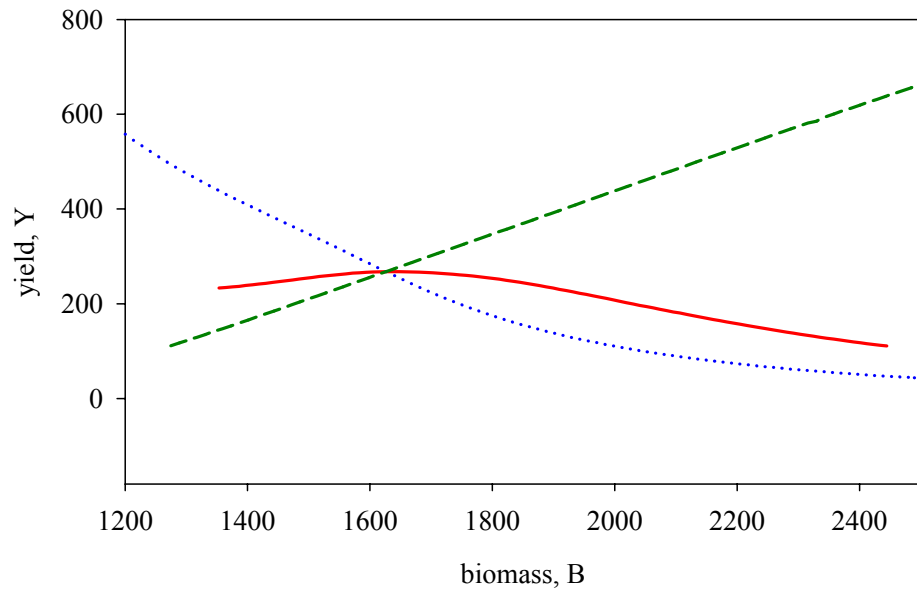


Figure 11. Optimal yield and size variations in different age classes

- Variation in age class 1
- - - Variation in age class 2
- ⋯ Variation in age class 3

Figures 10a,b show the outcome if the youngest (and smallest) fish are not commercially valuable. In Figure 10a, it is assumed that the three youngest age classes are valueless but that the catchability and fecundity parameters are as in the baseline case (Table 1). As shown in Figure 10a, the optimal solution does not converge to a steady state with constant yield and biomass. Instead, the long run optimal solution is a limit cycle where yield and biomass (and other variables) fluctuate over time. This type of solution is optimal, since after closing the fishery for some periods, the age distribution changes and a larger proportion of the catch will consist of the valuable older age classes. This solution therefore attempts to avoid growth overfishing<sup>8</sup>. If it is assumed that the gear has the knife edge selectivity property and  $q_s = 0$ ,  $s = 1, \dots, 3$ , the cycle disappears and the solution approaches a steady state with constant yield and biomass over time.

The concavity properties of the utility function also have implications for the cyclicity of the optimal solution. In Figure 10b, the path that stabilizes at the yield level of 200 thousand tons is the optimal solution if the concavity parameter in the utility function is decreased from 0.7 to 0.5. Thus, the cycle disappears when the concavity of periodic outcome with respect to yield is increased. In contrast, the dotted line in Figure 10b is the optimal solution when the objective function is linear with respect to yield (and effort). This solution is an example of a pure pulse fishing strategy.

Pulse fishing is a somewhat mysterious topic in fishery literature. Clark [8] offers an analysis of pulse fishing in the context of the classical Beverton-Holt model with constant exogenous recruitment, zero harvesting cost and linear utility. In his solutions, the entire fish population is harvested at regular intervals the length of which is solved using the Faustmann forest formula. Clark [8] writes that in his analysis exogenous recruitment is one prerequisite for pulse fishing.

Compared to Clark's analysis, the age-structured model presented here includes endogenous recruitment and nonlinear utility. With strictly concave utility, the difference to the "pure" pulse fishing solution analyzed by Clark [8] is that here the cyclical solution is an example of an interior limit cycle where the population and harvest levels remain strictly positive. The intuition is that since gear is nonselective, and the youngest age classes are not commercially valuable, the normal sustainable harvest strategy that is constant over time would imply growth overfishing. This can be partly avoided if harvesting is periodically decreased close to zero with the result that older age classes would form a larger proportion of the population. Under linear utility, the optimal harvest level is temporarily equal to zero between the periods with positive yield, but due to endogenous recruitment, the population level must remain strictly positive over the cycle.

#### *Optimal yield and number of recruits*

Horwood and Whittle [18] solve their model by linearizing the necessary conditions at a steady state, with the implication that the optimal aggregate yield becomes a linear function of fish in various age classes. Their interesting

result was a numerical example where the optimal aggregate yield decreases with recruits or the number of fish in the youngest age class. Figure 11 shows a similar analysis for the model studied here. The Figure is based on an example where,  $f_1 = f_2 = 0$ ,  $U = (10Y_t)^{0.7}$ ,  $b = 0.97$ ,  $C = E_t$  and production functions are of the Schaefer type. The x-axis shows the aggregate biomass when the number of fish in age-class one, two or three is varied and other age classes are kept fixed at their steady state levels. The y-axis shows the optimal aggregate yield for the first period. As shown, the aggregate yield initially increases and later decreases as a function of the recruit biomass. Next, it can be observed that the optimal yield is a monotonically decreasing and convex function of the fish biomass in the second age-class. For age classes three (and older), the optimal yield increases with the size of the given age class.

The result that the optimal yield may decrease with the aggregate biomass if the increase is in the youngest age classes must be contrasted with the surplus production model where the optimal yield is always a non-decreasing function of biomass. The age structured approach recognizes that the population age structure contains valuable information on future harvesting possibilities and that it is economically optimal to take this information into account in the present harvesting decisions. In the present case, it is optimal to postpone effort and harvest the large cohort later when its weight and economic value is higher. Figure 11 also suggests that linearization procedures may lose some essential properties of the optimal solution.

## 6 Summary

This study has analyzed optimal harvesting of a fish population when the analysis is based on a generic age-structured ecological model. This approach can be viewed as a generalization of the economic model based on the surplus production approach. The results of this study suggest that the generalization changes several main properties of the optimal harvesting solutions. These changes can be summarized as follows:

1. In the surplus production model, sustainable yield is based on biological properties of the natural production function, while in the age-structured approach it is based both on biological factors and on fishing technology.
2. Excluding coincidences, the optimal steady states of the two models are equal only under zero rate of interest and when the price of fish is independent of its age or size.
3. Transition paths toward the steady states coincide only accidentally and are, in general, qualitatively different. In the surplus production framework the paths are monotonic but in the age-structured framework the paths (in biomass-yield state space) are typically nonmonotonic and may contain damped oscillations.
4. For populations with many age classes, the surplus production model may yield "optimal extinction" results under a too low rate of interest.
5. In the surplus production model, optimal yield is an increasing function of biomass, while in the age-structured framework, optimal yield is a func-

tion of the number of individuals in different age classes. If the population is weighted toward young age classes, the optimal yield may decrease in biomass.

6. If the optimization problem is linear in yield and effort, the surplus production model yields constant escapement. This policy is not optimal for the age-structured model.

7. Under knife edge selectivity, the equilibrium biomass-sustainable yield relationship does not exist in the usual sense and the optimal steady state may be (locally) independent of the rate of interest.

8. Depending on the catchability profile, on effort cost and price determination of fish, the age-structured model may yield limit cycles and pulse fishing as the optimal solution. Similar factors behind the pulse fishing strategy are difficult (or impossible) to analyse applying the surplus production model.

Some earlier studies, such as Hannesson [14] and Clark [8], have found that the age-structured model would almost inevitably yield the pulse fishing strategy as the optimal solution. This study suggests that even when the problem is linear in yield and effort the optimal solution may represent sustainable harvesting over a wide range of parameter values. To some extent, this difference may follow from the choice of this study to apply the normal approach in economic models where control is specified as an instantaneous event instead of the classical Beverton and Holt [2] formulation where fishing is constant over each period.<sup>9</sup>

The ongoing discussion on the applicability of the biomass approach [34,31,24,32] suggest that including the population age structure into economic studies deserves much more emphasis. An example of an open problem is a model with endogenous choice of fishing gear with different catchability profiles. In spite of the complexity it should be possible to study the age-structured model analytically without deviations from the most fruitful specifications. Analytical work will increase the understanding of why the optimal steady states differ for the two models. Studying deterministic fishery models should be understood only as stepping stones toward empirically more realistic models with stochastic recruitment and fish price. Adding multiple species or spatial structure should not produce overly complex problems for numerical analysis.

#### Footnotes:

<sup>1</sup> The surplus production model, (dynamic) biomass approach, lumped parameter model and the Schaefer [30] approach all refer to a specification where the fish stock dynamics is given by  $x_{t+1} = x_t + F(x_t) - h_t$ , where  $x_t$  is biomass,  $F$  is the growth function (e.g. logistic) and  $h_t$  is the rate of harvest.

<sup>2</sup>It is required that yield is a piece-wise linear increasing function of population biomass.

<sup>3</sup>Recall that in discrete time economic models the state is normally given in the beginning of each period and the control (e.g. consumption) at the end of each period. Previous optimization studies [18,31] have applied the original Beverton and Holt [2] specification, where effort is constant over each period. It is not straightforward to view this specification as being more general than

the other alternatives. This detail can be made more accurate by dividing the fishing season (normally a year) into shorter periods.

<sup>4</sup>Equal natural mortality over the age classes is typical in age-structured data. An extension of this would not cause any problems in optimization models.

<sup>5</sup>The fact that, for age structured populations, fishing technology is a factor that determines the equilibrium biomass-yield function is not circumvented when the surplus production model is directly estimated from empirical data on the harvested population (for an introduction to the estimation methods, see [13]).

<sup>6</sup>Note that due to complementary constraints, a potential nonlinearity exists if the solution does not remain in regime  $y_{st}^1 = 0$ ,  $s = 1, \dots, n$ ,  $t = 0, 1, \dots$

<sup>7</sup>The discount factor  $b = 0.8$  implies an annual rate of interest equal to 25%. This high interest rate level is chosen only to reveal theoretical properties of the optimal solution. Recall that in analytical work model properties may be studied when  $r \rightarrow \infty$ .

<sup>8</sup>In growth overfishing, fish are caught when they are considered to be too small and young. In recruitment overfishing, the spawning stock is harvested at a too low level.

<sup>9</sup>Recall that this specification is applied by several fishery ecologists as well [25,34,26].

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